Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Information Theory and Coding
ENEE 532
Final Exam

Instructor: Dr. Wael Hashlamoun   Date: May 25, 2013

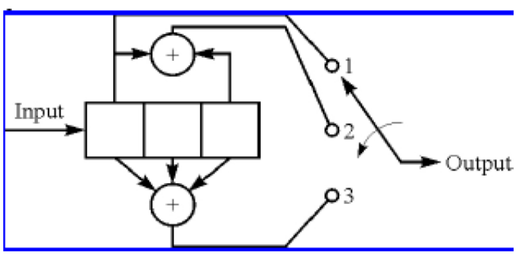
Problem 1: 22 Points
The generator matrix of a linear binary block code is

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

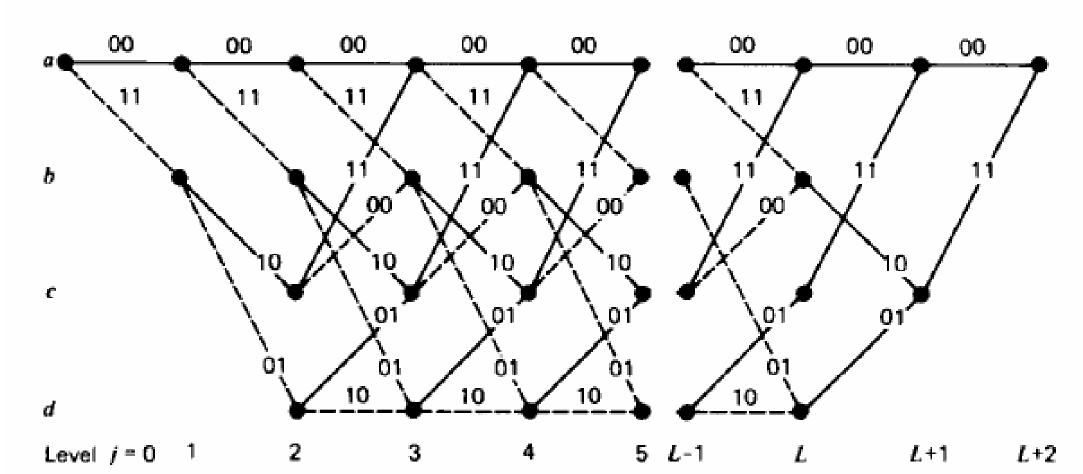
a. Find the codeword corresponding to the message **m** = (1 0 1 0)
b. Find the code rate
c. Find the corresponding parity check matrix, H, for this code.
d. Construct the syndrome table for this code.
e. If the received word is **r** = (1 1 1 0 0 0 1), find via syndrome decoding the codeword selected by the decoder and the corresponding message at the decoder output.

Problem 2: 18 Points
We want binary codes with length n = 255 and capable of correcting up to and including t = 3 errors.
a. According to the sphere packing bound, what is the minimum number of parity bits needed to achieve this error correcting capability?
b. How many erroneous bits can this code detect?

Problem 3: 22 Points
Consider the convolutional encoder depicted in Figure 1.
a. Find the rate of the code
b. Find the code corresponding to the message 10100
c. Construct the trellis diagram for the encoder.



Problem 4: 18 Points
The trellis diagram of a convolutional encoder is shown in Figure 2.



Use the Viterbi algorithm to decode the received sequence 10 10 00 01

Remark: A solid line means a “0” input, while a dashed line means a “1” input.

Problem 5: 20 Points

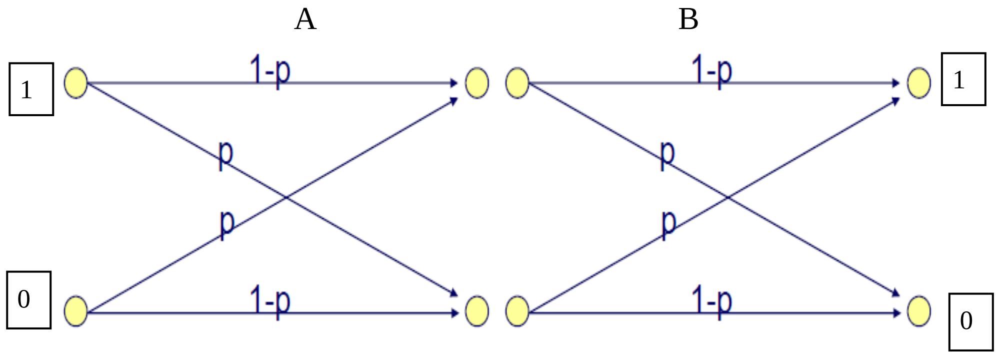
Two binary symmetric channels A and B are connected in cascade as shown in the Figure 3 below

a. The capacity of the binary symmetric channel is given as

$$C = 1 + p\log_2 p + (1-p)\log_2(1-p) = 1 - H(p)$$

Use this formula to find the capacity for p = 0.25.

b. By reducing the cascade into a single channel H, find the capacity of the new channel when p = 0.25.



Good Luck

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Information and Coding Theory ENEE 5304
Midterm Exam

Instructors: Dr. Wael Hashlamoun Date: April 11, 2018

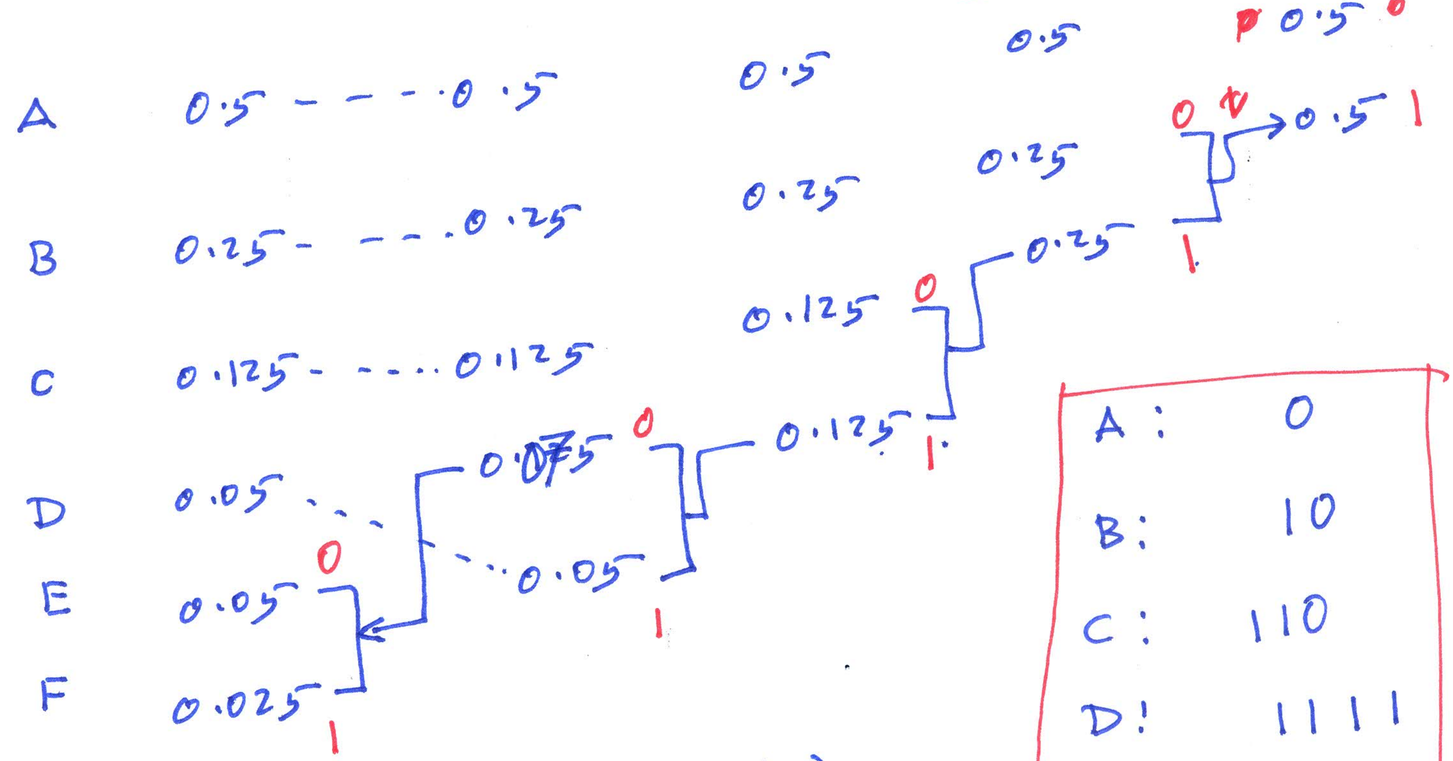
Problem 1: 22 Points

A discrete memory-less source produces six possible symbols with the following probabilities:

Symbol	A	B	C	D	E	F
Probability	1/2	1/4	1/8	1/20	1/20	1/40

5 a. Find the source entropy.
12 b. Find a binary Huffman code for the source.
5 c. Find the average number of binary digits per source symbol for the Huffman code found in part b.

$$H = -0.5\log_2 0.5 - 0.25\log_2 0.25 - 0.125\log_2 0.125 - 0.05\log_2 0.05 - 0.05\log_2 0.05 - 0.025\log_2 0.025 = \boxed{1.94 \text{ bits/symbol}}$$



A : 0
B : 10
C : 110
D : 1111
E : 11100
F : 11101

$$\bar{L} = (0.5)(1) + (0.25)(2) + 0.125(3) + (0.05)(4) + (0.05)(5) + (0.025)(5)$$

$$\boxed{\bar{L} = 1.95 \text{ bits/codeword}}$$

Problem 2: 22 Points

A discrete memory-less source produces 7 possible symbols with the probabilities given in the table below. Also, given in the table is one possible code.

Symbol	A	B	C	D	E	F	G
Probability	1/2	1/4	1/8	1/16	1/32	1/64	1/64
Code	0	10	110	1110	11110	111110	111111

5 a. Find the amount of information (in bits) produced by symbol E.

5 b. Find the average amount of information in a message consisting of eight symbols produced by the source.

4 c. Does there exists a prefix-free code with an average length smaller than that given in the table? Explain

4 d. If a fixed length code is used, find the required number of binary digits per source symbol.

4 e. Find the minimum achievable average number of binary digits per symbol source if a fixed length code is used.

a. $I_E = -\log_2 p(E) = -\log_2 \frac{1}{32} = 5$ bits

b. $H = -\sum p_i \log_2 p_i = \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \left(\frac{1}{16}\right)(4) + \frac{1}{32}(5) + \frac{2}{64}(6) = 1.968$ bits/symbol

$\boxed{H = 1.968 \text{ bits/symbol}}$

$\Rightarrow I = 8H = 8(1.968) = \boxed{15.75 \text{ bits}}$

c. $\bar{L} = (1)\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{16}\right) + 5\left(\frac{1}{32}\right) + 6\left(\frac{1}{64}\right) + 6\left(\frac{1}{64}\right)$

$\boxed{\bar{L} = 1.968}$

since $\boxed{\bar{L} = H}$ $\Rightarrow$ This is the smallest average code length.

d. $n = 3$ bits (7 symbols $\Rightarrow$ 3 bits)

e. $L_{min} = \log_2 7 = 2.807$ bits/symbol.

Problem 3: 20 Points

Give the Lempel-Ziv parsing and encoding of the binary data sequence 0100000000110101101010101. Here, you need to find the different sentences in the dictionary and their respective code words.

0, 1, 00, 000, 0001, 10, 101, 1010, 10101,

1	0	(0, 0)	00000
2	1	(0, 1)	00001
3	00	(1, 0)	00010
4	000	(3, 0)	00110
5	0001	(4, 1)	01001
6	10	(2, 0)	00100
7	101	(6, 1)	01101
8	1010	(7, 0)	01110
9	10101	(8, 1)	10001

Problem 4: 20 Points

Let X and Y be two independent random variables with the following marginal probability mass functions:

$$P(X) = \begin{cases} 1/3, & x = 1 \\ 1/3, & x = 2 \\ 1/3, & x = 3 \end{cases} \qquad Q(Y) = \begin{cases} 5/10, & y = 1 \\ 3/10, & y = 2 \\ 2/10, & y = 3 \end{cases}$$

7 a. Find the mutual information $I(X;Y)$ between X and Y.

7 b. Define the product of X and Y as: $Z = XY$. Find $H(Z)$, the entropy of Z.

6 c. Find the relative entropy (divergence) between X and Y defined as:

$$D(X,Y) = \sum_{i=1}^{3} p_i log_2\left(\frac{p_i}{q_i}\right)$$

a. $I(x;y) = \sum_x \sum_y p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)}$

since x and y are independent, then $p(x,y) = p(x)p(y)$

$\Rightarrow I(x;y) = \sum_x \sum_y p(x)p(y) \log_2 \frac{p(x)p(y)}{p(x)p(y)} \boxed{= 0}$

b. $H(Z) = H(x) + H(y)$; due to independence

$H(x) = \left(-\frac{1}{3}\log_2 \frac{1}{3}\right)3 = \log_2 3 = 1.584$ bits/symbol

$H(y) = -\frac{5}{10}\log\frac{5}{10} - \frac{3}{10}\log_2\frac{3}{10} - \frac{2}{10}\log_2\frac{2}{10} = 1.485$ bits/symbol

$\Rightarrow \boxed{H(Z) = 3.069}$

c. $D(x;y) = \frac{1}{3}\log_2\frac{0.333}{0.5} + \frac{1}{3}\log_2\frac{0.3333}{0.3} + \frac{1}{3}\log_2\frac{0.333}{0.2}$

$\boxed{D(x;y) = 0.1001}$

Birzeit University
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Department of Electrical and Computer Engineering
Information and Coding Theory ENEE 5304
Quiz # 1

Key

Instructors: Dr. Wael Hashlamoun Date: February 27, 2019

Problem
A discrete memoryless source emits one of the six symbols every time unit with the following probabilities:

a	b	c	d	e	f
0.36	0.18	0.18	0.15	0.08	0.05

a. Find the amount of information contained in symbol a
b. Find the amount of information contained in the message (a, e)
c. Find the source entropy H

a. $\boxed{I(a) = \log_2 \frac{1}{P_a}} = \log_2 \frac{1}{0.36} = \log_2 2.777 = \frac{\ln 2.777}{\ln 2} = \boxed{1.473 \text{ bits}}$ 1

2.5

b. $I(a \wedge e) = \boxed{I(a) \overset{1.5}{+} I(e)}$

2.5

$= \log_2 \frac{1}{0.36} + \log_2 \frac{1}{0.08} = \frac{1}{\ln 2}(\ln 2.777 + \ln 12.5)$ 1

$= (1.473 + 3.643) = \boxed{5.116 \text{ bits}}$

c. $H = \boxed{(-) \sum P_i \log_2 P_i}$

5

$= \frac{-1}{\ln 2}\{+0.36 \ln 0.36 + 0.18 \ln 0.18 + 0.18 \ln 0.18$
$+ 0.15 \ln 0.15 + 0.08 \ln 0.08 + 0.05 \ln 0.05\}$ 1

$= \frac{1.621}{\ln 2} = \boxed{2.339 \text{ bits/symbol}}$

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Quiz # 2

Instructors: Dr. Wael Hashlamoun Date: April 24, 2019

Problem 1: Find the capacity of the binary symmetric channel when $P(1|0) = P(0|1) = 0.1$

$C = 1 + [p \log p + (1-p) \log (1-p)]$ 1

$= 1 + \frac{[0.1 \ln 0.1 + 0.9 \ln 0.9]}{\ln 2}$ 1 1 1 $= 1 - \frac{(0.23 + 0.094)}{\ln 2}$

5/10

$= 1 - 0.467$ 5

$= 0.532$ bits / symbol (transmission) 1

Problem 2:
Find the capacity of a continuous channel with a bandwidth of 3.3 KHz and signal to noise ratio of 40 dB.

$40 \text{ dB} = 10 \log SNR \Rightarrow \log(SNR) = \frac{40}{10} = 4$ 1 1

$\Rightarrow SNR = 10^4 = 10{,}000$ 1 3

5/10

$C = W \log_2 (1 + SNR)$ 1 2

$= (3.3) \times 10^3 \log_2 (1 + 10{,}000)$

$C = 43.84$ k bits/sec 1

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Quiz # 3

Instructors: Dr. Wael Hashlamoun   Date: May 20, 2019

Problem
Consider the (6, 3) linear block code.

a. Can this code correct a single bit in error? Verify your answer
b. How many different codewords does this code generate? Justify
c. Can we select 000001 as a codeword? Explain

a. Use Hamming Bound

4 $2^k \sum_{j=0}^{t} \binom{n}{j} \leq 2^n \Rightarrow 2^3\left(1+\binom{6}{1}\right) \leq 2^6$

$2^3(7) \leq 2^6 \Rightarrow 56 \leq 64$

YES, it can correct a single error.

b. 3 $k=3 \Rightarrow 2^3 = 8$ codewords

c. NO, since its Hamming distance from the 0 code is 1 bit.

3 $d_{min} \geq 3$ since the code can correct single errors.

Problem 2: 20 Points

A discrete memory-less source produces one of 7 possible symbols every time unit with the probabilities given in the table below. Also, given in the table is one possible code.

Symbol	A	B	C	D	E	F	G
Probability	1/2	1/4	1/8	1/16	1/32	1/64	1/64
Code	0	10	110	1110	11110	111110	111111

5 a. Find the source entropy in bits/symbol.
4 b. Find the average number of bits/codeword.
3 4 c. Does there exists a prefix-free code with an average length smaller than that given in the table? Explain

3 4 d. Is it possible to reduce the average number of bits/symbol by combining two symbols together to form one message? Explain

4 e. If a fixed length code is used, find the minimum achievable average number of binary digits per symbol source.

a. $H = \sum p_i \log_2 p_i = \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 (2)^2 + \frac{1}{8}\log_2 2^3 + \frac{1}{16}\log_2 2^4 + \frac{1}{32}\log_2 2^5 + 2\cdot\frac{1}{64}\log_2 2^6$

$= \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{16}(4) + \frac{1}{32}(5) + \frac{2}{64}(6)$

a. $= 1.968$ bits/symbol

b. $\bar{L} = \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{16}(4) + \frac{1}{32}(5) + \frac{1}{64}(6) + \frac{1}{64}(6)$

$\bar{L} = 1.968$ bits/codeword

c. Since $\bar{L} = \bar{H} \Rightarrow$ This is the smallest possible average codeword

d. No, we cannot go beyond H.

e. Minimum fixed length encoding $= \log_2 M$

$= \log_2 7$

$= 2.807$ bits/symbol

Problem 3: 20 Points

Consider the binary sequence:

$$0100000000110101101010101$$

Find the Lempel-Ziv code corresponding to this sequence. Here, you need to find the different sentences in the dictionary and show their respective code words.

8X2.5 = 20

position	Dictionary	(position, new) code format	codeword
1	0	(0,0)	0000 0
2	1	(0,1)	0000 1
3	00	(1,0)	0001 0
4	000	(3,0)	0011 0
5	0001	(4,1)	0100 1
6	10	(2,0)	0010 0
7	101	(6,1)	0110 1
8	1010	(7,0)	0111 0
9	10101	(8,1)	1000 1

Problem 4: 20 Points

Let X and Y be two random variables related through the following joint probability mass function:

		0.3	0.34	0.36
	X \ Y	**0**	**1**	**2**
0.16	**0**	0.1	0.06	0
0.34	**1**	0	0.28	0.06
0.5	**2**	0.2	0	0.3

6

a. Find the entropies $H(X)$ and $H(Y)$.

b. Find the relative entropy (divergence) between X and Y defined as:

$$D(X,Y) = \sum_{i=1}^{3} p_i log_2\left(\frac{p_i}{q_i}\right)$$

c. Under what conditions can the relative entropy be negative?

a. $H(X) = -\sum p_i \log_2 p_i = -\frac{1}{\ln 2}\{0.16 \ln 0.16 + 0.34 \ln 0.34 + 0.5 \ln 0.5\}$

3 $\boxed{H(X) = 1.452}$

$H(Y) = -\sum p_j \log_2 p_i = -\frac{1}{\ln 2}\{0.3 \ln 0.3 + 0.34 \ln 0.34 + 0.36 \ln 0.36\}$

3 $\boxed{H(Y) = 1.5808}$

b. $D(X;Y) = 0.16 \log_2 \frac{0.16}{0.3} + 0.34 \log_2 \frac{0.34}{0.34} + 0.5 \log_2 \frac{0.5}{0.36}$

$= \frac{1}{\ln 2}\{-0.1005 + 0 + 0.1642\} = 0.0919$

4 $\boxed{D(X;Y) = 0.0919}$

c. $D(X;Y) \geq 0$

4 cannot be negative

It can be 0 when $p_i = q_i$; $i = 1,2,3$

but not -ve.

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Information and Coding Theory ENEE 5304
Final Exam

Instructors: Dr. Wael Hashlamoun   Date: June 4, 2017

Problem 1: 18 Points
A discrete memoryless source emits one of the following symbols every time unit with the given probabilities

Letter	*Probability*
A	1/2
B	1/4
C	1/8
D	1/16
E	1/16

a. Construct an efficient, uniquely decodable binary code, having the prefix-free property and having the shortest possible average code length per symbol.
b. How do you know that your code has the shortest possible average code length per symbol?

Problem 2: 18 Points
Consider the data sequence 0 1 0 0 0 0 1 1 0 1 0 1 0 0 1 1 1 0 1 1, which will be encoded using the Limpel-Ziv algorithm
a. Parse the data into different phrases to create the dictionary
b. How many bits are needed to represent each phrase?
c. Find the codeword for each phrase

Problem 3: 22 Points
Given the generator matrix of a linear block code

$$G = \begin{bmatrix} 1\ 0\ 0\ 0\ 0\ 1\ 1 \\ 0\ 1\ 0\ 0\ 1\ 0\ 1 \\ 0\ 0\ 1\ 0\ 1\ 1\ 0 \\ 0\ 0\ 0\ 1\ 1\ 1\ 1 \end{bmatrix}$$

a. How many codewords can this code generate?
b. Find the codeword for the message (1000)
c. Find the associated parity check matrix H^T
d. Generate the syndrome table for single error correction
e. If the sequence 1100011 is received, use the syndrome table of Part d to find the correct codeword

Problem 4: 22 Points

Suppose a cyclic redundancy check (CRC) code uses the prime generator polynomial

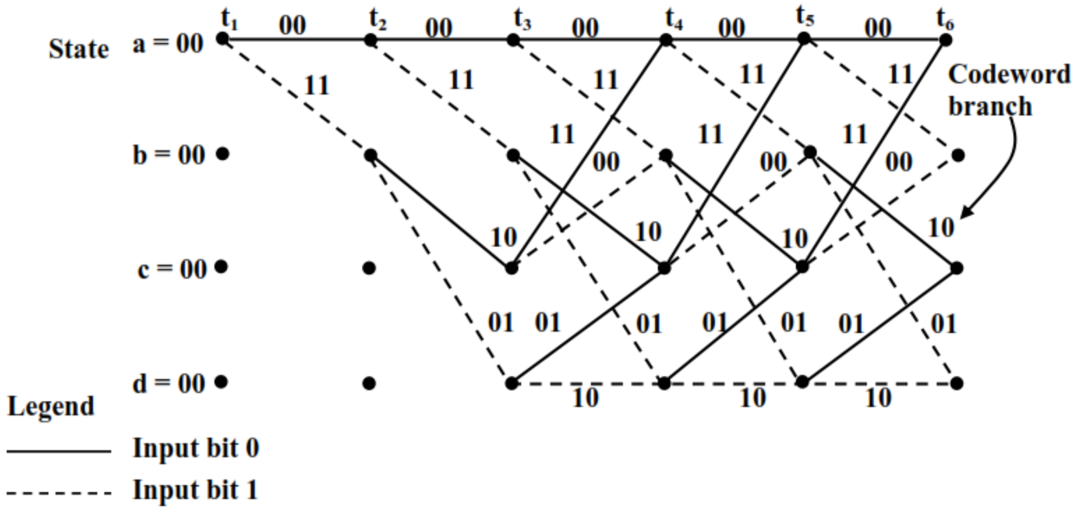
$$g(x) = x^3 + x + 1.$$

a. Generate the CRC bits for the message 1101
b. If the received sequence is 0001111, will the receiver accept is as a codeword?
c. If s(x) is the transmitted sequence, y(x) the received sequence, and e(x) the error sequence, then y(x) = s(x) + e(x). You know that: remainder (s(x)/g(x)) = 0. Use this information to find out if this polynomial is able to detect the error pattern 0001011 ? Verify
d. Can this CRC code detect a single error with a 100% certainty? Explain

Problem 5: 20 Points

The trellis diagram of a convolutional encoder is shown in the figure below.

a. If state a is 00, find states b, c, and d
b. Use the trellis diagram to find the codeword corresponding to the message 10100 assuming the encoder starts at the 00 state
c. Use the Viterbi decoding algorithm to find the most likely data sequence corresponding to the received sequence $(10,10,00,10,11)$



Good Luck

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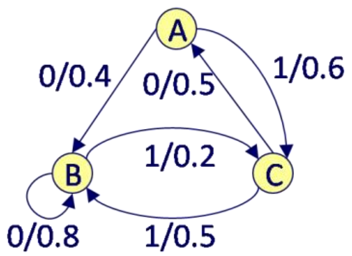
Information and Coding Theory ENEE 5304
Midterm Makeup Exam

Problem 1:
A stationary discrete Markov source can be in any one of three states, A, B, or C. When it is in any one of the states it emits either a 1 or a 0 with probabilities as shown in the figure below.



a. Find the steady state probabilities of the states A, B, and C
b. Find the source entropy.

Problem 2:
The joint probability mass function of two random variables X and Y is shown in the table below.

		Y	
		2	3
X	0	0.45	0.12
	1	0.15	0.28

a. Find H(X)
b. Find I(X; Y)